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DESIGN OF AXIAL PIN CONNECTIONS FOR TORQUE TRANSMISSION

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1. State of the Art

Axial pin connections consist of pins arranged axially around the perimeter as driving pins (**Fig. 1**). The pins can be varied with respect to number and arrangement.

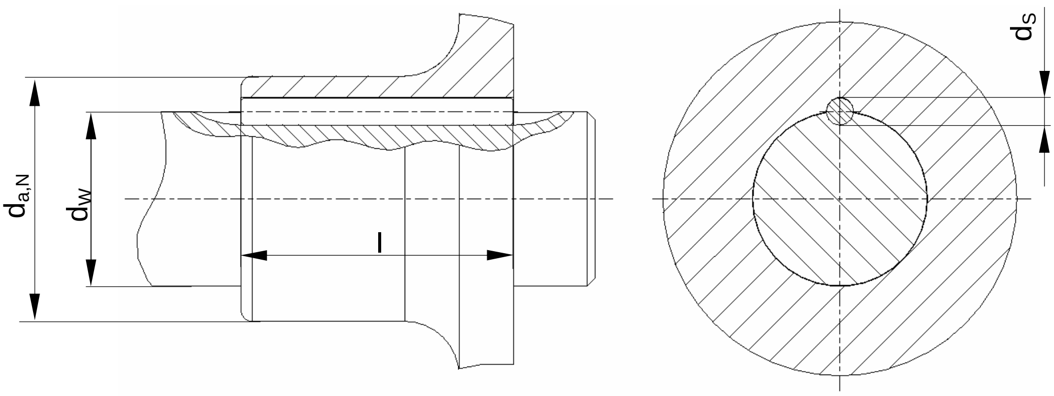

Figure 1. Longitudinal pin connection as driving pin in a shaft-hub-connection

There is no literature concerning the design of longitudinal pin connections with clearance [Faulwasser 1998]. It appears feasible to use the data for no-clearance connections as reference point. These design references are however few. The first dimensioning basis for the longitudinal pin connections with clearance was developed at the Institut für Maschinenwesen within the framework of a research project sponsored by the DFG.

2. Determination of Stress concentration factors

2.1 Stress concentration factors for Torsion

Due to the lack of knowledge concerning the form and fatigue strength of shafts and hubs weakened by a semicircular notch in the axial direction [Kollmann 1984], the work here concentrates on the determination of stress concentration factor diagrams for shafts and hubs with the help of the finite element method. The highest notch stress determined using the FE calculation is divided by the analytically calculated stress in the undisturbed cross-section:

$$a_K = \frac{\text{maximum stress by FE - Analysis}}{\text{nominal stress, analytical}} \tag{1}$$

This methodology was carried out for the shaft and hub-sided notch for different types of stress and variations in geometry:

- **shaft:** ratio of pin diameter to shaft diameter d_S/d_W; number of driving pins
- **hub:** ratio of outer hub diameter to shaft diameter $d_{a,N}/d_W$ at various d_S/d_W; number of driving pins

The stress was determined both analytically and using the FEM for an unattenuated geometry in order to clarify the mounting and load conditions. The chosen boundary conditions with reference to [Wesolowski 1994] resulted in a maximum deviation of < 1 % from the analytical calculations, i.e. an adequately high precision was guaranteed. The position of the pins was arranged so that the seam radius between the shaft and hub runs through the central point of the pin. The parameter calculations were then carried out using these boundary conditions. **Figure 2** shows the stress concentration factor diagram of the shaft for torsion.

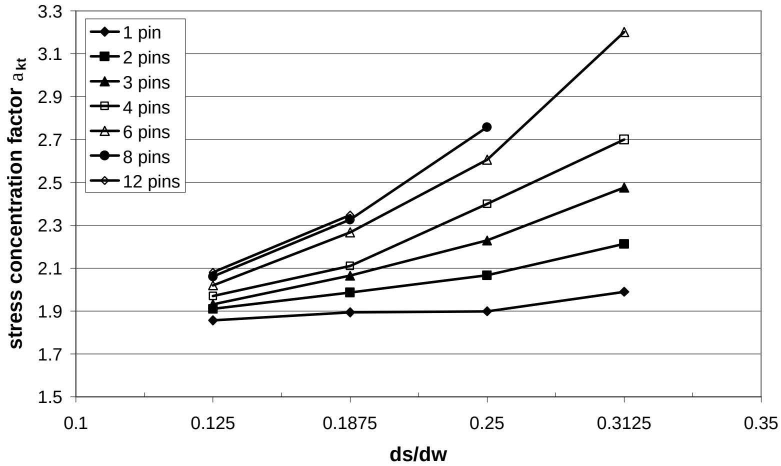

Figure 2. Stress concentration factor for the shaft at various d_S/d_W ratios and n pins

For 8 and 12 pins, no meaningful geometry resulted for d_S/d_W > 0.25 or 0.1875 so that the stress concentration factors could not be determined in this range. Equations for the analytical determination of the stress concentration factors were then derived from the curves.
It should be mentioned that although the notch radius increases with the ratio d_S/d_W and therefore reaches good values, an increase in the stress concentration factor results. This is due to the influence of the reduced load-bearing cross-sectional area which outweighs that of the increased notch radius.
Figure 3 shows the stress concentration factors determined for the hub at a ratio of $d_{a,N}/d_W$ =2.

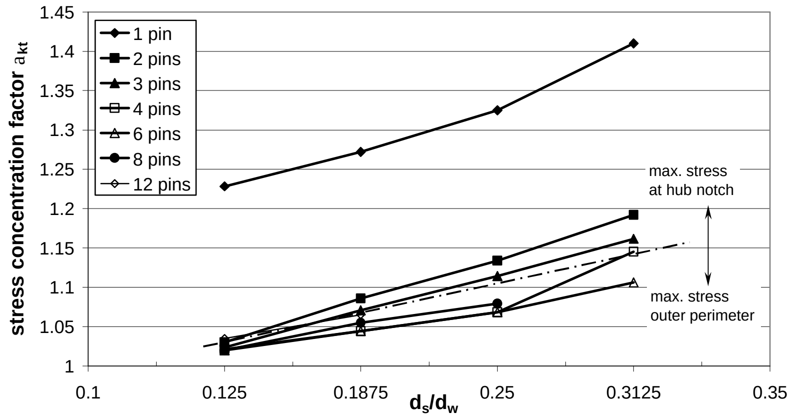

Figure 3. Stress concentration factor for the hub, $d_{a,N}/d_W$ = 2; various d_S/d_W ratios and n pins

The stress concentration factor diagram was also determined for the ratios $d_{a,N}/d_W$ = 1.5 and $d_{a,N}/d_W$ = 2.5. The further away a point in a hollow shaft is from the central axis, the greater the torsion stress at this point. Therefore, the maximum stress must not necessarily occur in the internal notch. The transitional range of maximum stress from the interior to the exterior is shown in Figure 3 in addition to the course of the stress concentration factors. At a ratio of $d_{a,N}/d_W = 1.5$, the maximum stress is independent of the number of pins and the ratio d_S/d_W is always in the range of the notch. At a ratio of $d_{a,N}/d_W = 2.5$, the maximum stress occurs at the outer diameter.

2.2 Stress concentration factors for Bending

The stress concentration factor diagrams for the bending strain are formulated using a procedure analogous to that for the determination of the stress concentration factors for torsion. **Figure 4** shows the stress concentration factor diagram for the shaft.

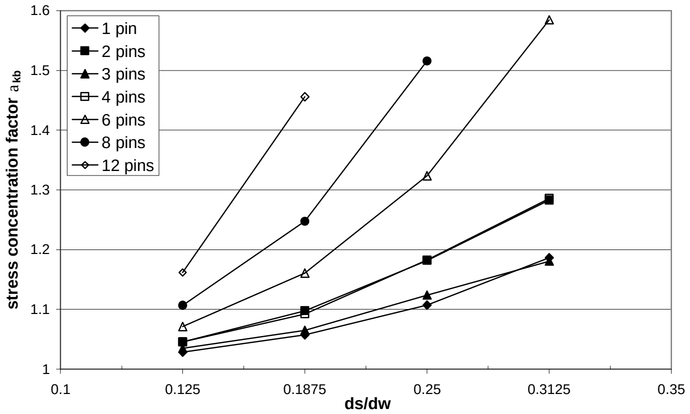

Figure 4. Bending stress concentration factor for the shaft for various d_S/d_W ratios and n pins

The interaction between the notch radius and the weakened cross section can also be observed for this type of stress.

Figure 5 shows the bending stress concentration factor diagram for the hub with a ratio of $d_{a,N}/d_W = 1.5$.

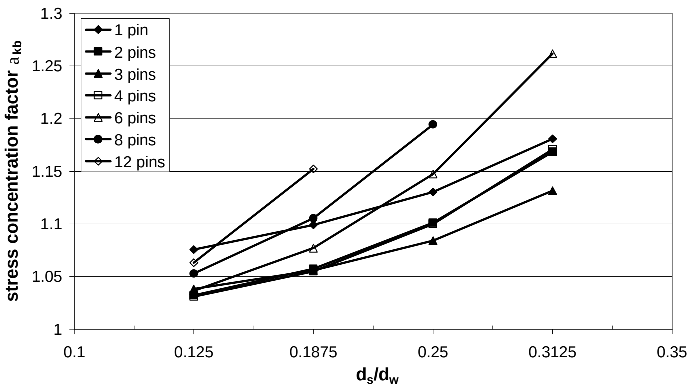

Figure 5. Bending stress concentration factor for the hub, $d_{a,N}/d_W = 1.5$; various d_S/d_W ratios and n pins

The values for $d_{a,N}/d_W = 2$ and $d_{a,N}/d_W = 2.5$ lie clearly beneath those shown in Figure 5. Already at the ratio $d_{a,N}/d_W = 2$, the maximum stress concentration factors occurring lie at $\alpha_{kb} = 1.07$, so that the use of a stress concentration factor $\alpha_{kb} = 1.1$ above a ratio of $d_{a,N}/d_W = 2$ for a stability test represents a conservative design.

3. Influence of the connection width on the load-bearing capacity

The few sources available give a ratio of connection length to shaft diameter l/d_W of 1...1.5 as approximate value for the dimensioning of a longitudinal pin connection [Niemann 1981] [Fronius 1987] [Wächter 1987]. However, extensive investigations of cog-shaft connections [Wesolowski 1997] show that at a ratio of $l/d_B > 0.6$ (l: connection width, d_B: reference diameter for spline shafts), the maximum stress resulting from the jump in stiffness at the start of the connection cannot be

reduced. The maximum stress drops steeply in the axial direction (higher stress gradient) and then flattens out to a low level until the end of the connection.
Since the geometry of a longitudinal pin connection suggests an analogy to a spline-shaft, the range $l/d_W = 0.5...0.75$ was examined more closely. **Figure 6** shows the influence of the connection length on the pin.

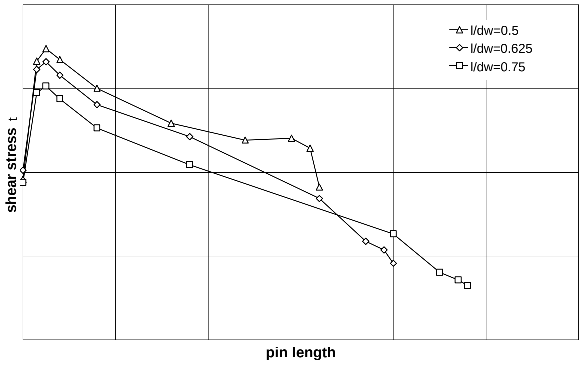

Figure 6. Influence of the connection width on stress for $d_S/d_W = 0.125$; $d_W = 32mm$, 4 pins

A distinct influence of the connection length on the pin can clearly be seen. The shearing stress could be reduced up to 16 % by using a longer connection. This supports the conclusion that a wider connection reduces the stress of the entire connection.
Figure 7 shows the course of the equivalent stress in the notch base of the hub. A reduction of the stress in the hub analogous and in the same order of magnitude as that in the shaft can be observed.

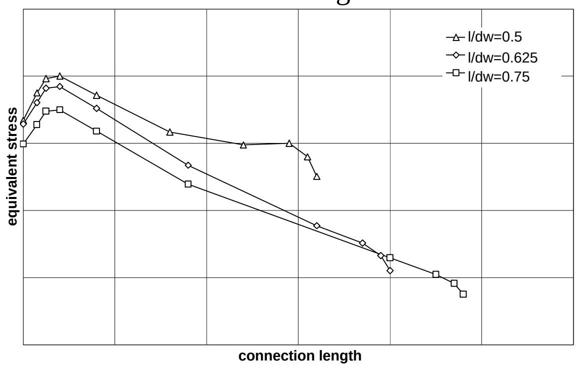

Figure 7. Influence of the connection width on the stress in the hub for $d_S/d_W = 0.125$, $d_W = 32mm$, 4 pins

Figure 8 shows the course of the equivalent stress for a shaft.

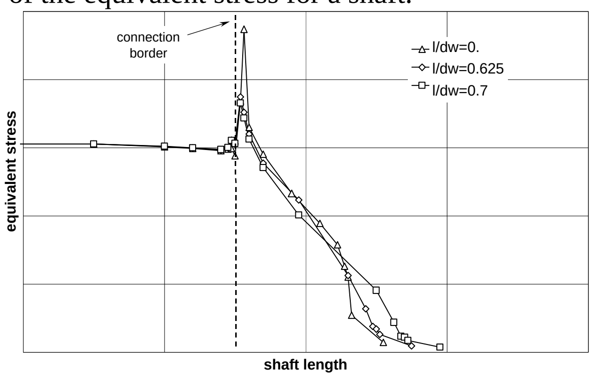

Figure 8. Influence of the connection width on stress in the shaft for $d_S/d_W = 0.125$, $d_W = 32mm$, 4 pins

A change in the connection width has a significantly greater effect on the shaft than the pin and the hub. A corresponding course of equivalent stress can be seen for all ratios l/d_W in the undisturbed section of the shaft, in other words before the beginning of the connection. A section with a distinct stress increase is present at the beginning of the connection, which exhibits an increased value of 60% opposed to the undisturbed section for the ratio l/d_W =0.5. A widening of the connection leads to a significant reduction of the overstress to 19%. At ratios above l/d_W = 0.625, it must be observed that no significant reduction in the overstressing occurs. Therefore, it can be recommended that a ratio of the connection width to shaft diameter of l/d_W = 0.6...0.7 should be maintained.

4. Influence of the pin diameter on the connection stress

A reduction of the shearing stress in the pin is to be expected due to the increase of the pin diameter and the corresponding increase of the shearing area. This is clearly shown in **Figure 9**. The shearing stress in the pin is significantly reduced.

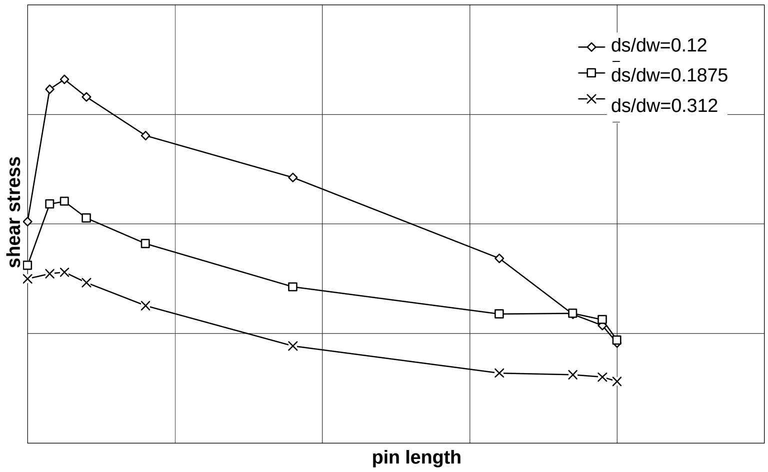

Figure 9. Influence of the pin diameter on the pin stress. 4 pins, d_W = 32; l/D_W = 0.625; $d_{a,N}/d_W$ = 2

The level of stress in the notch root is shown in **Figure 10.**

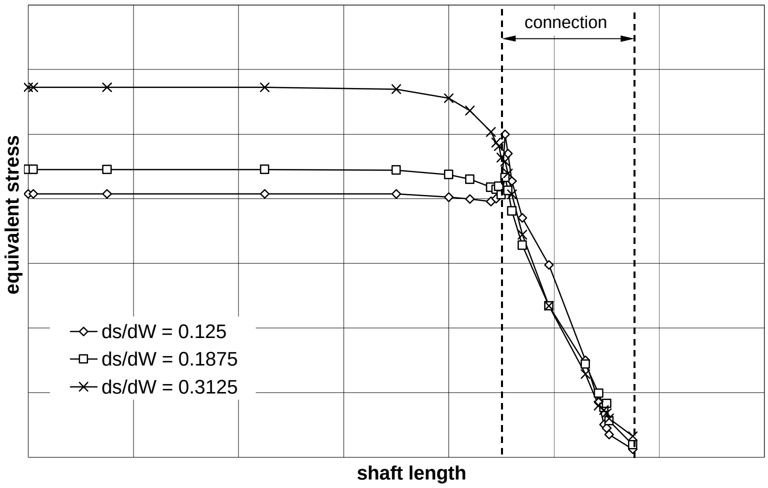

Figure 10. Influence of the pin diameter on the shaft stress. 4 pins, d_W = 32; l/D_W = 0.625; $d_{a,N}/d_W$ = 2

Due to the increase in the pin diameter, the stress peak at the beginning of the connection is reduced on the one hand and on the other, the cross-section of the shaft in simultaneously weakened causing the stress before the contact to increase significantly. With a pin diameter of d_S = 6 mm, which corresponds to a ratio of d_S/d_W = 0.1875, the stress before and in the contact zone are almost equal for a connection with 4 pins.

In contrast to the shaft, the equivalent stress in the hub sinks, whereby the changes are slight for a pin diameter of 6 mm, **Figure 11**. An increase in the stress is to be expected for larger pin diameters d_S>10 mm because the load-bearing cross-section is reduced.

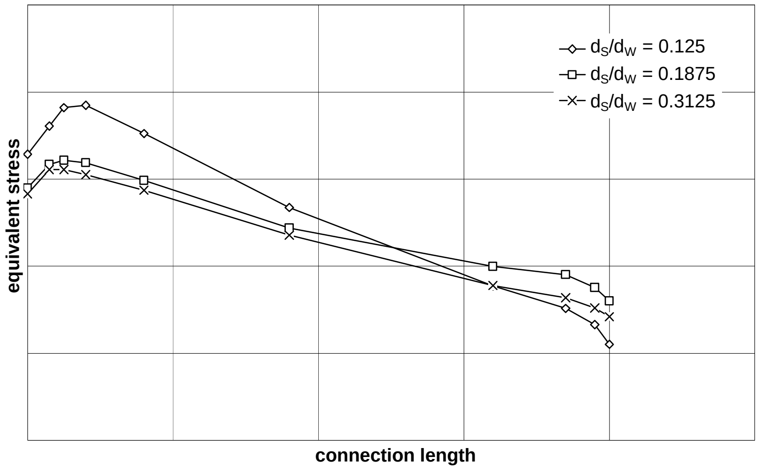

Figure 11. Influence of the pin diameter on the hub stress. 4 pins, d_W = 32; l/D_W = 0.625; $d_{a,N}/d_W$ = 2

5. Conclusion

Extensive parameter studies lead to the development of stress concentration factor diagrams for the shaft and hub for the torsion and bending stress for a shaft-hub connection with axially arranged pins. The influence of the different geometry variations could clearly be seen when the entire longitudinal pin connection system is taken into consideration. A protrusion of the pin at the beginning of the connection reduces the stress in the pin and shaft, the influence on the hub is only slight. The advantage over other stress-reducing measures, such as increasing the number of pins or the pin diameter, lies in the fact that neither the shaft nor the hub are weakened through a higher notch effect or that the manufacturing expenditure increases through additional notches and pins.

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