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IMROVING THE ROBUSTNESS OF MULTICRITERIA DECISION MAKING

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Abstract

Selecting among alternatives is a central and frequent decision in design. This selection is often done to satisfy multiple criteria and with imperfect information. This paper proposed a new method for handling extreme uncertainty in multicriteria decision-making based on Analytic Hierarchy Process (AHP) and info-gap models of uncertainty. The method foundation is briefly reviewed and it is illustrated on a simple problem. Subsequently, the paper formulates a new problem in decision-making with a proposed solution. Instead of asking how to exercise “good” (we say good because there is no best) selection practice (e.g., selection among alternative product concepts), the decision setup changes to include resources, and the question becomes: how to spend them optimally to improve information quality for making choices that are more robust. The new method is illustrated on a simple selection problem.

Keywords: unstructured uncertainty, resource-based decision making, information quality, selection

1 Introduction

Decision-making activities are always performed under uncertainty conditions. This uncertainty reflects the gap between perfect knowledge about the information we need to use in our decision-making process and what we actually know. The information we use is often *imprecise* and of varying levels of *reliability*. These two qualities contribute to the information *uncertainty*.

Selection among alternatives is perhaps the most prevalent decision making activity in general and in design specifically. Many models exist for this activity [5], however, their treatment of the reliability of the information they employ is lacking, complex, or requires assumptions on the nature of uncertainties (e.g., the distribution density of extreme uncertain events) [2]. We seek a very user-friendly model that its logic is easy to understand, is simple to use, and its results are easy to interpret. The model should be based on simple and minimal data and output the robustness of the decision and its reliability.

In addition, we introduce a novel aspect into the decision-making process: given a resolved decision problem with its associated reliability, and available resources; spend these resources optimally in order to maximize the decision robustness by improving the reliability of the information input to the decision-making process. In product design, such resources would be used to perform targeted studies of the most critical aspects of the new design: study new technology, build prototypes, etc.

We are not aware of any published study that connects decision robustness, risk, or uncertainty with resources allocated to reduce it. We describe in detail a method for addressing this problem, called RBR (Resource-Based Robustness). In the method exposition, we assume knowledge of AHP, otherwise, the reader is referred to [4], or the many others books and papers published on it.

2 Multicriteria decision making under extreme uncertainty

The proposed method is composed of the following ingredients:
1. AHP for modeling complex decision-problem by a hierarchy and for organizing evaluations [4],
2. Arbel's method for approximate preference articulation (instead of AHP eigenvector calculations) [1],
3. Ben-Haim's method for modeling uncertainty [2],
4. Linear programming for solving the equations and obtaining a decision, and
5. AHP for combining the contribution of different levels of the hierarchy.

We discuss each in turn.

2.1 Approximate articulation of preferences

In AHP, a decision-maker (DM) evaluates the candidates for selection using pairwise comparisons. Traditionally, the comparisons have been exact values. In an attempt to account for DMs lack of exact knowledge or the inherent uncertainty in the environment, Arbel described the use of approximate articulation of preferences $l_{ij} \le w_i / w_j \le u_{ij}$, where $l_{ij} \le u_{ij}$ for all $i, j = 1, 2, ..., n$ and w_i / w_j is the preference of item i in relation to j. These preferences can be assembled and written as

$$\begin{aligned} &Aw \le 0, \\ &w_1 + w_2 + \cdots + w_n = 1, \\ &w_1, w_2, \ldots, w_n \ge 0. \end{aligned} \tag{1}$$

Since all the solutions to the problem posed above are restricted to lie on the simplex $w_1 + w_2 + \cdots + w_n = 1$, the set of inequalities for a *feasible problem* form a convex region Ω on this simplex. For a 3-dimensional problem, an approximate articulation process will yield (at most) six inequalities, and hence this case may be envisioned as shown in Figure 1.

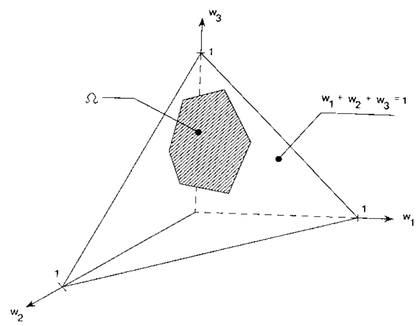


Figure 1: A solution set for a solvable system

The consistency index (CI) used in the AHP assumes the value zero when pairwise comparisons form a completely consistent set. Here, the completely consistent case is obtained when all half spaces intersect at a single point on this simplex. Since this will reduce Ω to a point, Arbel argues that the size of this area provides some measure of consistency.

A solution to the model in equation (1) can be found using linear programming (LP) by solving an auxiliary LP problem given by

$$\begin{aligned}
&\min w_0 \\
&\quad \text{subject to} \\
&Aw \leq 0, \\
&w_1 + w_2 + \cdots + w_n = 1, \\
&w_0, w_1, w_2, \ldots, w_n \geq 0.
\end{aligned} \tag{2}$$

where w_0 is an artificial variable used to identify the existence of a feasible solution. If the ordering of preferences is equal in all vertices of the convex hull Ω, it holds for each point inside the area.

Since we address decision problems that are more complex or hierarchical, the solution of Equation (2) is repeated for all the comparison matrices of AHP. Finally, AHP method for propagating preferences along the hierarchy is used here as well to find an area in the space of weights as shown in Figure 1. We can prove that the result we get by working with hierarchies is a convex region. Consequently, if the same option is selected as best in all the region vertices, it is also the preferred option across the region.

2.2 Info-gap model of uncertainty [2]

One useful classification of uncertain phenomena is to distinguish between structured and unstructured uncertainty: structured uncertainty is made of extensive data, from which a probability density function can be constructed for predicting mean standard deviation and other statistical properties. An unstructured uncertainty, on the other hand, is a total surprise, an unexpected event. Unstructured uncertainty is a substantial information-gap between what we do know and what we need to know in order to perform optimally [2].

When dealing with severe lack of information we need to be very parsimonious with the information that we have. Therefore, we must avoid unverifiable assumptions as much as possible. In particular, we will not be able to adopt probability densities, since it is the rare events that dominate our concern. Second, we cannot make statistical inferences, as though we were facing very structured and familiar uncertainty. Rather than probabilistic reliability, the *robustness is adopted as a measure of reliability.* We define robust reliability of a decision as the amount of uncertainty consistent with no alteration of the decision. We now define the model of uncertainty and a decision procedure with its associated robustness measure.

Consider the set of all functions $c(t)$ whose relative deviation from the nominal function $\tilde{c}(t)$ is bounded by $\alpha\psi(t)$:

$$C(\alpha, \tilde{c}) = \{c(t) : |\frac{c(t) - \tilde{c}(t)}{\tilde{c}(t)}| \leq \alpha\psi(t)\}, \alpha \geq 0 \tag{3}$$

where:

- $c(t)$ is the real function (unknown to us);
- $\tilde{c}(t)$ is the function assessed by us according to the information available;
- $C(\alpha, \tilde{c})$ is a set that contains all the functions consistent with our prior information;
- $\psi(t)$ defines a known envelope within which the function varies; and
- α is *the uncertainty parameter*.

The robustness of a decision is defined as the maximum value of α consistent with the desired value of the decision. Formally, if $D(q,c)$ is some procedure for deriving a decision based on the design variables q and the c. Then the robustness of the decision as a function of q would be the maximum value of α while keeping the decision intact.

$$\hat{\alpha}(q) = \max\{\alpha : D(q,c) = D(q,\tilde{c}) \text{ for all } c \in U(\alpha,\tilde{c})\} \tag{4}$$

In our case, the decision procedure is solving Equation (1) (or (2)) while requiring that the same best alternative (i.e., the choice having the highest w_l) is the same in the feasible region Ω. The parameter c would denote the ranges of preferences described in Section 2.1. We use a discrete version of Equation (3) leading to:

$$\tilde{c}_{ij} - \psi_{ij}\tilde{c}_{ij}\alpha \le c_{ij} \le \tilde{c}_{ij} + \psi_{ij}\tilde{c}_{ij}\alpha\ , \tag{5}$$

or, considering the range $\lfloor l_{ij}, u_{ij} \rfloor$ we get,

$$\begin{aligned} u_{ij} &= \tilde{c}_{ij} + \psi_{ij}\tilde{c}_{ij}\alpha \\ l_{ij} &= \tilde{c}_{ij} - \psi_{ij}\tilde{c}_{ij}\alpha \end{aligned} \tag{6}$$

where

l_{ij} ,u_{ij} are lower and upper bounds respectively, their difference reflecting the *impreciseness* of the information,

$\tilde{c}_{ij} = w_i / w_j$ is the assessment of the relative weight of option i relative to option j, and

ψ_{ij} is the assessed reliability of the $\tilde{c}_{ij}$ parameter (or range).

Therefore, the decision procedure consists of:

1. For a given α and set of values ψ_l, $l = 1,\ldots,n$, solve Equation (1) (or (2)) with values derived from Equation (6).
2. Find the minimal value, $\alpha_{\min}$ for which there is a feasible solution (the area Ω is a point).
3. Find the maximal value of α, $\alpha_{\max}$ for which there is a feasible region Ω, and the best option with highest w_l, remains the same across all Ω.

The value $\alpha_{\min}$ is needed to account for inconsistent preference articulation and has similar role as that of CI or CR in AHP. To illustrate the function of $\alpha_{\min}$ in some problem, Figure 2 shows a progression from (a) infeasible solution ($\alpha = 0$), to (b) (c) where "rays" (representing the constraints in Equation (1)) coming out of the corners begin to separate due to the growing imprecision. Finally, at a value of $\alpha = 1.111$ the areas between the rays meet to start forming a feasible region shown in (d). For the same preferences, AHP outputs: CI= 0.61559, and CR= 1.0614.

The $\alpha_{\max}$ is an indication of the robustness of the decision. Higher values of $\alpha_{\max}$ mean that we can be less precise about our information yet, the decision will not change. Higher values of $\alpha_{\max}$ mean that the decision is more immune to uncertainty. When the information is precise, the α parameter will be set to zero. Lower precision will be expressed by higher α values. An α value of one or more will denote a considerable imprecision. After using the method for some time, the DM will recognize the meaning of the resulting α on the outcome of decisions, and learn to consider it accordingly. The numbers ψ_{ij} will be used to express the *reliability* of the information source. As in the case of the preciseness parameter, experience will tell the DM the influence of ψ_{ij} values on the solutions. In case the reliability is unknown and assessed to be no better than any other source, then ψ_{ij} will be set to one.

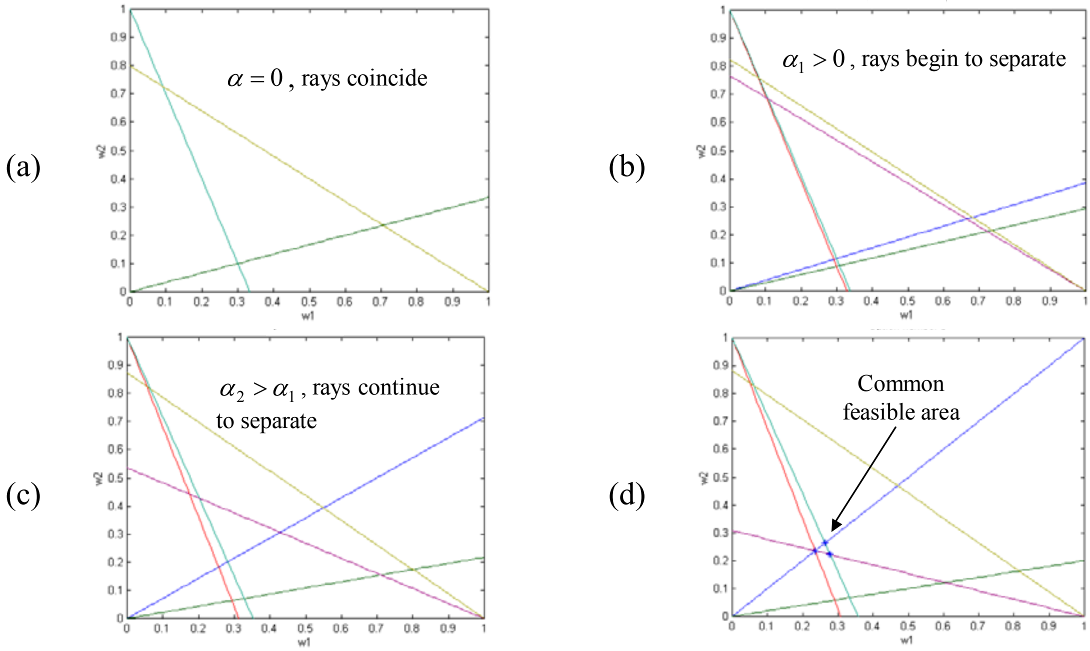
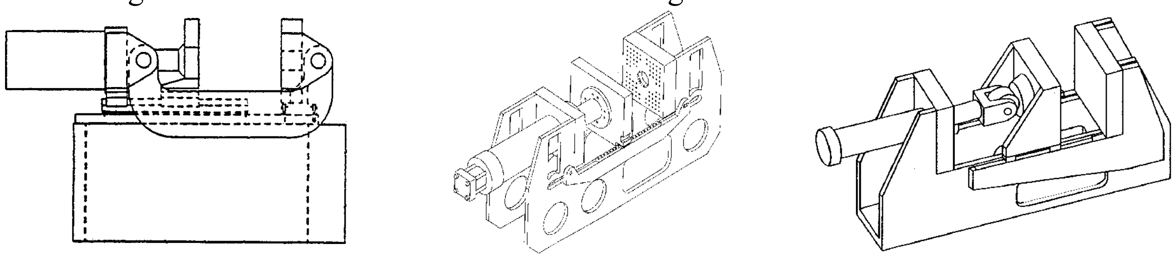


Figure 2: Obtaining feasible area by increasing α beyond α_{min}

3 Example

We illustrate the method using a problem of selecting between three plastic injection-molding machines. The selection is done between three tiebarless[1] machines shown in Figure 3. These machines have all been patented; nevertheless, for the sake of the example, consider them as three concepts that a design team is evaluating for further detailed development. The decision is broken into three main criteria: lifetime, simplicity of parts, and simplicity of maintenance. Their arrangement in the form of AHP is shown in Figure 4.



C_1 Engel's tiebarless C_2 Ziv-Av's, elastic deflection C_3 Ziv-Av's, no aligning rod

Figure 3: Three candidate tiebarless injection molding machines

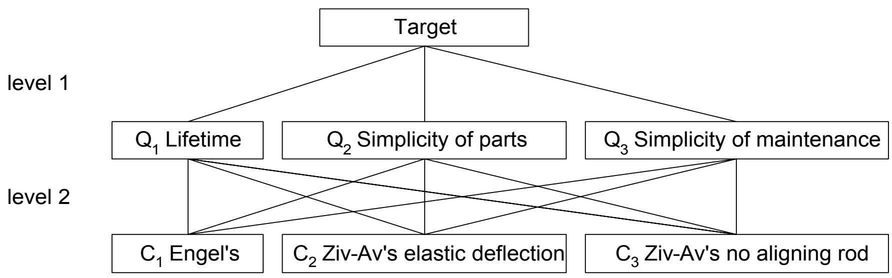


Figure 4: Hierarchy of criteria used to choose a plastic molding machine

[1] Machines without the common ties guiding the parallel movement of the moving platen.

The first and second hierarchy level pairwise comparison values (the relative importance of the qualities) elicited from an expert designer are given in Figure 5 and Figure 6, respectively. From experience, it is easier to elicit preferences values ($\tilde{c}_{ij}$) and reliability measures (ψ_{ij}) instead of ranges ($\lfloor l_{ij}, u_{ij} \rfloor$). Therefore, these values are given in the figures. The values of the reliabilities were set to 0.1 for all evaluations. Below each pairwise comparison matrix, the values of the largest eigenvalue, the CI, and CR are given.

All the CR are within the limits recommended by AHP (lower than 0.1), even though in the second quality comparison (simplicity of parts) the ratio is close to the limit. This consistency affects the minimal preciseness value $\alpha_{\min}$ required for a viable solution, which as expected is high: $\alpha_{\min}$ =3.1, compared to α =1, which symbolizes a value with a "regular" precision.

		Q_1	Q_2	Q_3
Lifetime	Q_1	1	5	2.5
Simplicity of parts	Q_2	1/5	1	1/3
Simplicity of maintenance	Q_3	1/2.5	3	1

λ_{max}=3.0183 CI=0.00914 CR=0.0157

Figure 5: First hierarchy level comparison values

	Lifetime	C_1	C_2	C_3
Engel	C_1	1	1/3	1/9
Elastic	C_2	3	1	1/6.5
No rod	C_3	9	6.5	1

λ_{max} =3.067 CI=0.033 CR=0.058

Simplicity of Parts	C_1	C_2	C_3
C_1	1	4	½
C_2	4	1	1/3
C_3	2	3	1

λ_{max}=3.108 CI=0.054 CR=0.093

Simplicity of Maintenance	C_1	C_2	C_3
C_1	1	6	1
C_2	1/6	1	1/6
C_3	1	6	1

λ_{max} =3 CI=0 CR=0

Figure 6: Second hierarchy level comparison values (the comparison between the options, pertinent to each quality)

Table 1: Expanded ranges for $\alpha_{\max}$ =8.15

The Ratio Compared	Comparison #		Input value $\tilde{c}_{ij}$	Expanded ranges
First hierarchy	1	Q1/Q2	5	[0.925 9.0751]
	2	Q1/Q3	2.5	[0.4625 4.5375]
	3	Q2/Q3	1/3	[0.0616 0.605]
Q_1: Lifetime of the machine	4	C1/C2	1/3	[0.0736 0.605]
	5	C1/C3	1/9	[0.0205 0.1689]
	6	C2/C3	1/6.5	[0.0339 0.2792]
Q_2: Simplicity of parts	7	C1/C2	4	[0.74 7.2602]
	8	C1/C3	1/2	[0.0925 0.9075]
	9	C2/C3	1/3	[0.0616 0.605]
Q_3: Simplicity of maintenance	10	C1/C2	6	[1.11 10.8904]
	11	C1/C3	1	[0.185 1.815]
	12	C2/C3	1/6	[0.0308 0.3025]

At one vertex of the feasible region, the preferences are 0.136, 0.090, and 0.774, making option C_3 the best. This is the best option across the feasible domain and remains as such until the value of $\alpha_{\max}$ =8.15. While this says little intuitively at this point, we can calculate the pairwise ranges according to Equation 6 and compare them to the original values, see Table 1. For example, comparisons 1 and 10 extend even beyond the 1-9 range of AHP saying that if other comparisons are as in the Table, it does not matter which value the DM gives for these comparisons; they will lead to the same choice. The other ranges are also quite large suggesting that this decision is quite robust to uncertainties. Note that the ranges such as [0.0205 0.1689] are also large being reciprocals of a large range, i.e., [1/48 1/5.9].

4 RBR – Maximizing robustness under resource constraints

The previous section dealt with fixed given information reliabilities. However, in reality, DMs have resources to spend in order to collect information for improving the quality of their information and decisions. Given limited budget, a DM needs to choose those information sources that have the most impact on the robustness of the decision, and distribute the funds optimally between them. The DM could have two different goals:

1. Maximize decision making reliability/robustness given fixed budget, or
2. Minimize cost of improving information quality for attaining a prescribed robustness.

In this paper, we address only the first goal. The method called Resource-Based Robustness maximization (RBR) includes two additional steps beyond those of the regular method (see Section 2):

6. Reich and Levy's method for resource allocation in product development [3], and
7. Goal programming for modeling and solving the optimal resource allocation problem.

In order to develop the method we have to construct a function that connects between resources spent and resulting information reliability. In addition, we will have to construct a measure of decision robustness since the $\alpha_{\max}$ value might not be directly applicable. These issues are addressed below.

One-way to improve a decision robustness in the best way possible is to find comparison limits that we can contract, i.e., improve their certainty, which will enable us to expand the other limits as much as possible. Since we are improving the robustness of the decision, this manipulation of the limits must maintain the best choice.

It might be that no budget is needed to expand the feasible area. Since we start the optimization with $\alpha_{\max}$ that is common to all matrices, it might be possible that in a certain matrix we can still expand the limits of the input without affecting the final outcome of the calculation. The system can calculate the expansion reached in the final feasible area by expanding the limits of the comparison values in one of the matrices and checking that the chosen option remains the same.

But besides such free expansion, in general, in order to expand one pairwise comparison, we would have to contract another. This would require investing resources in a specific uncertain comparison value to improve its certainty. Another method to improve the solution robustness would be to improve the reliability of a specific source of information that will also narrow a specific comparison limit, thus allowing the expansion of others. Since the quality (in our case the uncertainty) is not linearly improved as a function of the efforts invested, we need a model to simulate the investment needed to reach a certain uncertainty level. The model used was taken from [3].

In order to improve the accuracy of our assessment, we will try to raise the lower limit and to lower the higher limit obtained. In the case of raising the original lower limit of the value (marked l_{ij}^{o}) to a new value l_{ij}^{n}, the following model describes the relationship between the new lower limit of the value and the investment needed to reach it:

$$g(l_{ij}^{n}) = \begin{cases} 0 & l_{ij}^{n} < l_{ij}^{0} \\ -1/t * \log\left(\dfrac{1-(l_{ij}^{n}-l_{ij}^{0})}{u_{ij}^{0}-l_{ij}^{0}}\right) & l_{ij}^{n} \geq l_{ij}^{0} \end{cases} \tag{7}$$

where:

l_{ij}^{o} – the original lower limit of the value,

u_{ij}^{o} – the original upper limit of the value,

l_{ij}^{n} – the new lower limit,

$g(l_{ij}^{n})$ – the investment needed to reach l_{ij}^{n}, and

$$t = -1\Big/A_{b\%j} \cdot \log\left(\frac{100-b_{\%}}{100}\right) \tag{8}$$

where $A_{b\%j}$ – the investment needed in comparison j to reach $b_{\%}$ of its maximal comparison range thus creating a perfect and certain comparison. Fixing this value is the task of the DM. It can be done based on experience and may evolve during decision-making.

This relationship describes the investment needed $(g(l_{ij}^{n}))$ as a function of the new limit l_{ij}^{n}. The maximal improvement is obtained by reaching the other limit, in this case, u_{ij}^{o}. A similar function can be written for lowering the upper limit:

$$g(u_{ij}^{n}) = \begin{cases} 0 & u_{ij}^{n} > u_{ij}^{o} \\ -1/t * \log\left(\dfrac{1-(u_{ij}^{o}-u_{ij}^{n})}{u_{ij}^{o}-i_{ij}^{o}}\right) & u_{ij}^{n} \leq u_{ij}^{o} \end{cases} \tag{9}$$

The functions in Equations 8 and 9 are shown in Figure 7. In the calculation process described later, the investment of moving the lower limit is first calculated and then the investment of improving the higher limit is calculated.

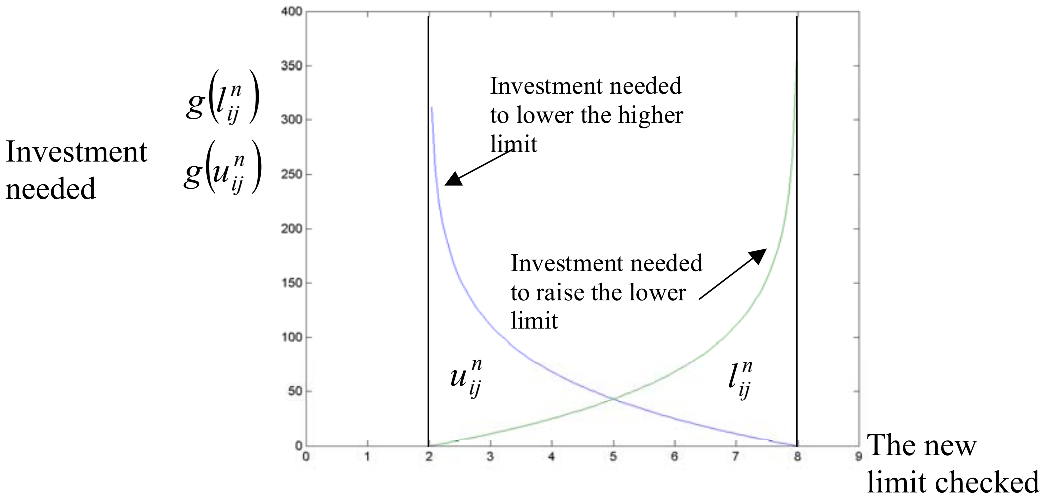


Figure 7: The investment needed as a function of the initial limits, the maximal limit possible and the desired new value of the limit

The robustness of the decision was modeled using the area of the feasible domain Ω. Although according to Ben-Haim, $\alpha_{\max}$ denotes the robustness of the decision, once we allow different $\alpha_{\max}$ values, we cannot use it as the required measure. Consequently, we use a less theoretically motivated measure but one the intuitively is meaningful: large area means more latitude in assigning preferences while maintaining the same decision. In the future, we would study this measure and its relation to $\alpha_{\max}$ in detail.

We formulate the robustness maximization problem as follows:

$$\begin{aligned}
&\max Area(\Omega) = f\left(l_{ij}^{o,m}, u_{ij}^{o,m}, l_{ij}^{n,m}, u_{ij}^{n,m}\right) \\
&\text{subject to:} \\
&\sum_{m=1}^{M}\left(g\left(l_{ij}^{n,m}\right) + g\left(u_{ij}^{n,m}\right)\right) \leq B \\
&1/9 \leq l_{ij}^{n,m}, u_{ij}^{n,m} \leq 9
\end{aligned} \tag{10}$$

The $Area(\Omega)$ is calculated by following the algorithm in Section 2 for obtaining a feasible area. The total spending on comparison range contraction (increase of $l_{ij}^{o,m}$ and decrease of $u_{ij}^{o,m}$) is bounded by the total budget B. The additional upper index m denotes the index of the comparison matrix in the hierarchical problem model (in the example problem in Section 3, there are 4 matrices). The problem in Equation (10) has been solved by goal programming.

5 Example revisited

In Section 3, we obtained a robustness measure of selecting between three plastic molding machines. With a global value of $\alpha_{\max}$=8.15, the solution was quite robust. In order to demonstrate the RBR method, we let the ranges of the criterion “Simplicity of parts” vary in response to some investment. The 3D space in Figure 8 shows the extent to which we can improve the robustness of the overall solution by investing 500-unit budget (e.g., men hours) in improving the comparison values of the “Simplicity of parts” comparison matrix. The darker line surrounding the smaller region shows the feasible area before the investment, and the lighter lines show the feasible area after the investment. It can be seen that small decrease in some comparisons led to large increase in another. Consequently, the solution robustness is significantly enlarged.

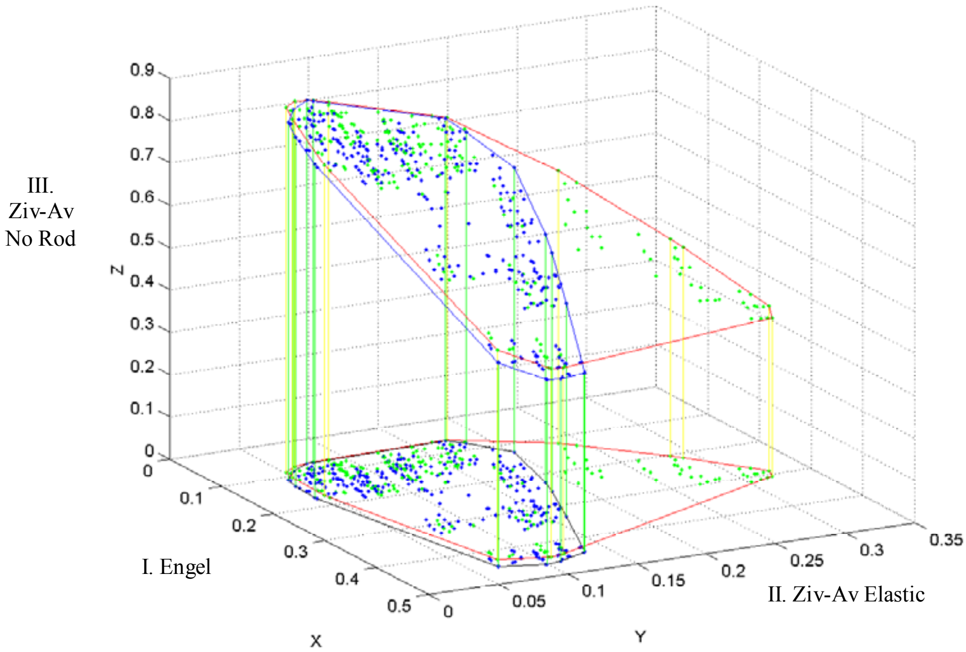


Figure 8: Drawing of the feasible area and its reflection at x-y plane

The next question to ask is: what is the budget that we should invest? If this budget is too small, we might not achieve the potential robustness and remain with a possible unstable answer. In contrast, we have to be sure that the entire budget invested is used properly and not wasted. In order to find what is the optimal budget to be used, the influence of the budget investment on the robustness was studied. The size of the feasible area as a function of the budget used was calculated (the size of the feasible area is the parameter used to check the changes in the robustness). It was found that beyond 1000 unit budget, the solution robustness did not change. Therefore, this is the maximally required budget.

6 Discussion

This paper presented a method for calculating the robustness of decision-making scenarios. This method can assist the task of selecting between alternatives so profound in engineering design. The method rests on simple models with minimal assumptions on the nature of uncertainty. In addition, the paper presented a new problem in decision-making with its solution: maximizing the robustness of decision by utilizing resources. The solution to both problems was demonstrated on a simple but realistic decision problem. While there are other studies in the literature that deal with decision making with range or fuzzy preferences, and some that deal with choices with partial information, we are not aware of a study dealing with the second problem. In the future, we intend to develop these methods further, to better understand the meaning of the parameters of the method, and to explore different resource allocation functions and robustness decision measures.

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